## FYS 3610

## EXERCISES WEEK 43

## EXERCISE 1

a) Sketch a vertical profile of electron density versus altitude for noon sunlit conditions, and also for midnight non-auroral conditions. Indicate D, E, and F-regions on your sketch.

The ion and electron drifts perpendicular to the magnetic field can be written as:

$$
\begin{align*}
& \overrightarrow{\mathrm{v}}_{\mathrm{i} \perp}=\frac{\mathrm{k}_{\mathrm{i}}}{1+\mathrm{k}_{\mathrm{i}}^{2}} \frac{\overrightarrow{\mathrm{E}}_{\perp}}{\mathrm{B}}+\frac{\mathrm{k}_{\mathrm{i}}^{2}}{1+\mathrm{k}_{\mathrm{i}}^{2}} \frac{\overrightarrow{\mathrm{E}}_{\perp} \times \overrightarrow{\mathrm{B}}}{\mathrm{~B}^{2}}  \tag{Eq. 2.1}\\
& \overrightarrow{\mathrm{v}}_{\mathrm{e} \perp}=-\frac{\mathrm{k}_{\mathrm{e}}}{1+\mathrm{k}_{\mathrm{e}}^{2}} \frac{\overrightarrow{\mathrm{E}}_{\perp}}{\mathrm{B}}+\frac{\mathrm{k}_{\mathrm{e}}^{2}}{1+\mathrm{k}_{\mathrm{e}}^{2}} \frac{\overrightarrow{\mathrm{E}}_{\perp} \times \overrightarrow{\mathrm{B}}}{\mathrm{~B}^{2}} \tag{Eq. 2.2}
\end{align*}
$$

where $\mathrm{k}_{\mathrm{i}}=\frac{\Omega_{\mathrm{i}}}{v_{\mathrm{in}}}$ and $\mathrm{k}_{\mathrm{e}}=\frac{\Omega_{\mathrm{e}}}{v_{\mathrm{en}}}$. (The neutral wind has been neglected).


Figure 2.1
b) Given Figure 2.1 discuss the rotation of $\overrightarrow{\mathrm{v}}_{\mathrm{i}}$ and $\overrightarrow{\mathrm{v}}_{\mathrm{e}}$ by height as well as their variation in amplitude by height from 50 to 200 km .
c) Based on Eqs. 2.1 and 2.2 derive expressions for the height-integrated Hall and Pedersen currents. Suggest reasonable boundary values on the height integrals. Justify your answer.

Assume an Earth-fixed Cartesian coordinate system $(x, y, z)$ where $x$ is pointing magnetic northward, $y$ magnetic eastward, and $z$ downwards towards the Earth's center. In this coordinate system the magnetic field is given by

$$
\begin{equation*}
\overrightarrow{\mathrm{B}}=\mathrm{B}(\cos \mathrm{I} \hat{\mathrm{x}}+\sin \mathrm{I} \hat{\mathrm{z}}) \tag{Eq. 2.3}
\end{equation*}
$$

and the electric field is given by

$$
\begin{equation*}
\overrightarrow{\mathrm{E}}=\mathrm{E}_{\mathrm{x}} \hat{\mathrm{x}}+\mathrm{E}_{\mathrm{y}} \hat{\mathrm{y}}+\overrightarrow{\mathrm{E}}_{\mathrm{z}} \hat{\mathrm{z}} \tag{Eq. 2.4}
\end{equation*}
$$

For this coordinate system it is assumed that the magnetic dipole axis is antiparallel to the Earth's rotation axis and that the magnetic field is symmetric around this axis. In this coordinate system the height-integrated current can be expressed on tensor form as:

$$
\left[\begin{array}{l}
\mathrm{J}_{\mathrm{x}} \\
\mathrm{~J}_{\mathrm{y}} \\
\mathrm{~J}_{\mathrm{z}}
\end{array}\right]=\left[\begin{array}{ccc}
\Sigma_{\mathrm{P}} \sin ^{2} \mathrm{I}+\Sigma_{\|} \cos ^{2} \mathrm{I} & -\Sigma_{\mathrm{H}} \sin \mathrm{I} & \left(\Sigma_{\|}-\Sigma_{\mathrm{P}}\right) \sin \mathrm{I} \cos \mathrm{I} \\
\Sigma_{\mathrm{H}} \sin \mathrm{I} & \Sigma_{\mathrm{P}} & -\Sigma_{\mathrm{H}} \cos \mathrm{I} \\
\left(\Sigma_{\|}-\Sigma_{\mathrm{P}}\right) \sin \mathrm{I} \cos \mathrm{I} & \Sigma_{\mathrm{H}} \cos \mathrm{I} & \Sigma_{\mathrm{P}} \cos ^{2} \mathrm{I}+\Sigma_{\|} \sin ^{2} \mathrm{I}
\end{array}\right]\left[\begin{array}{l}
\mathrm{E}_{\mathrm{x}} \\
\mathrm{E}_{\mathrm{y}} \\
\mathrm{E}_{\mathrm{z}}
\end{array}\right]
$$

d) Apply Eq. 2.5 on the equatorial region where the inclination angle is zero and $\vec{B}=B \hat{x}$. Assume that the vertical current $\mathrm{J}_{z}=0$ and show that

$$
\begin{equation*}
\mathrm{J}_{\mathrm{y}}=\left(\Sigma_{\mathrm{P}}+\frac{\Sigma_{\mathrm{H}}^{2}}{\Sigma_{\mathrm{P}}}\right) \mathrm{E}_{\mathrm{y}} \tag{Eq. 2.6}
\end{equation*}
$$

which is the equatorial electrojet. Comment on the analogy with the auroral electrojet.

## From Kievelson\&Russell:

## Exercise 14.1

Exercise 14.2

Exercise 14.8

